|  |  |  |
| --- | --- | --- |
| **Prob #** | **Points** | **Max** |
| **a** |  | **10** |
| **b** |  | **10** |
| **c** |  | **10** |
| **d** |  | **15** |
| **e** |  | **15** |
| **f** |  | **15** |
| **g** |  | **15** |
| **h** |  | **10** |
| **Total** |  | **100** |

A 200 m diameter, 15 cm length cylindrical hollow fiber tube is lined with a solute with MW 785 and equilibrium concentration 0.3 moles/L. The solute is plasma at a temperature of 37 C, with dynamic viscosity 0.012 g/(cm-s) and density 0.98 g/ml. The flow rate in the fiber is 0.76 ml/min (cross-sectional averaged velocity is 40 cm/s).

1. Use the Reynolds number to determine whether the flow can be considered to be fully developed in the fiber.

The Reynolds number is

The entry length is

So we can consider the flow to be fully developed.

1. Find the diffusion coefficient for the solute in plasma.

From the empirical model

If you decide to use Stokes-Einstein

1. Find the Schmidt number for this solute in plasma

If you used the empirical formula for

If you used Stokes-Einstein

1. Use the Reynolds number and Schmidt number to determine if the concentration boundary layer can be considered to be fully developed or “short contact time”.

The required length for a fully developed concentration profile is (empirical)

Or (Stokes-Einstein)

Because this length is an order of magnitude longer than the tube (in both cases), we can use short contact time.

1. Use the appropriate Sherwood number (from the table at the end of this exam) to determine the mass transfer coefficient.

The Sherwood number for fully developed flow and short contact time is

Therefore

Empirical

Stokes-Einstein

1. Assume (initially) that you do not need to worry about using the log mean concentration. Determine the concentration of the solute as it exits the tube.

The mass per unit time entering from the tube wall must equal the mass per unit time exiting the tube. With minimal buildup of solute in the solvent

So

Or, for Stokes-Einstein

1. State whether your answer to Part f indicates that the log mean concentration should be used in this case.

Because the concentration at the exit is far less than the equilibrium concentration, it is not necessary to use the log mean concentration.

1. Your friend has told you that the concentration profile for this flow has the form

with .

Use Fick’s law and the definition of the mass transfer coefficient to find the expression for the local mass transfer coefficient at axial position for this condition.

We have

When you evaluate the gamma function (which I do not expect you to do) you get

The book gives in terms of diameter instead of radius, so the coefficient in the correlation is times 0.855

**Potentially Useful Formulas**

**Differential Equations**

|  |  |
| --- | --- |
|  |  |
| Fourier Equation | or |
| Similar to 4, but with negative linear term | or |

Let be a characteristic velocity, be a characteristic length, be kinematic viscosity,

Further, let be diffusion coefficient.

Finally, let be the mass transfer coefficient.

**General mass transport equation**

In Cardesian coordinates

In cylindrical coordinates

In spherical coordinages

**Boundary layer development**

Pipe flow, fully developed momentum boundary layer .

Pipe flow, fully developed concentration boundary layer .

Flat plate, laminar boundary layer .

Cylinder, laminar flow .

**Constants**

Avagadro’s Number:

Faraday’s Constant:

Universal Gas Constant:

Centigrade to Kelvin: Degrees Kelvin 273.15 Degrees Centigrade

**Table of Laplace Transforms**

|  |  |  |
| --- | --- | --- |
| 1 |  | 1 |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 23 |  |  |
| 24 |  |  |
| 25 |  |  |
| 26 |  |  |
| 27 |  |  |

**Sherwood Numbers**

|  |  |
| --- | --- |
| Condition | Sh |
| Sphere in a stagnant fluid | 2 |
| Forced convection over a sphere |  |
| Laminar flow over a flat plate |  |
| Laminar flow in a cylindrical tube, short contact time |  |
| Laminar flow in a cylindrical tube, fully developed flow and concentration profiles | 3.66 |
| Turbulent flow through a circular tube |  |
| Spinning Disk |  |
| Falling Film, Average |  |